Steps for Analyzing the Graph of a Rational Function R:

1. **Factor** the numerator and denominator of R and find its domain. If 0 is in the domain, find the *y*-intercept, R(0) and plot it.
2. **Write R in lowest terms** as $\frac{p(x)}{q(x)}$ **and find the real zeros of the numerator**, i.e. set $p(x) = 0$. These are the *x* intercepts of the graph. Determine the behavior of the graph of R near each *x* intercept using the same procedure as polynomial functions. Plot each *x*-intercept and indicate the behavior of the graph near it.
3. **With R in lowest terms** as $\frac{p(x)}{q(x)}$ and **find the real zeros of the denominator**, i.e. set $q(x) = 0$. These are the vertical asymptotes of the graph. Graph each vertical asymptote using a dashed line.
4. **Locate any horizontal or oblique asymptotes** using the criteria below. Graph the asymptotes using a dashed line. Determine the points, if any, at which the graph of R intersects these asymptotes. Plot any such points
   a. **If the degree of the numerator is less than the degree of the denominator**, R is a proper rational function and the graph of R will have the horizontal asymptote $y = 0$.
   b. **If the degree of the numerator is greater than or equal to the degree of denominator**, then R is improper and we use ***long division***.
      i. **If the degree of the numerator equals the degree of the denominator**, the quotient obtained will be the number a_n/b_m (the coefficients of the highest power of the polynomial) and the line $y = a_n/b_m$ is a horizontal asymptote.
      ii. **If the degree of the numerator is one more than the degree of the denominator**, the quotient obtained is of the form $ax + b$ and the line $y = ax + b$ is an oblique asymptote.
      iii. **If the degree of the numerator is two or more than the degree of the denominator**, the quotient obtained is a polynomial of degree 2 or higher, and R has neither a horizontal nor an oblique asymptote. In this case, for $|x|$ unbounded, the graph of R will behave like the graph of the quotient.
5. Using the real zeros of the numerator and denominator of the given equation for R, **divide the *x*-axis into intervals and determine where the graph is above and below the *x*-axis** by choosing a number in each interval and evaluating R there. Plot the points found.
6. **Analyze the behavior of the graph of R near each asymptote** and indicate the behavior on the graph.
7. Put all the information together to **obtain the graph of R**.

YAY! YOU ARE DONE WITH *ONE* PROBLEM!!!!

Use the Steps 1-7 to analyze the graph of each function.

1.

$$F(x) = \frac{x^2 + 3x + 2}{x - 1}$$

2.

$$R(x) = \frac{8x^2 + 26x + 15}{2x^2 - x - 15}$$